Matematik

Potens: $(-a)^{2n} = +a^{2n}$; $(-a)^{2n+1} = -a^{2n+1}$; $a^m \cdot a^n = a^{m+n}$; $a^m : a^n = a^{m-n}$; $a^m \cdot b^m = (ab)^m$;

$a^m : b^m = \left(\frac{a}{b}\right)^m$; $\frac{1}{a^m} = a^{-m}$; $(a^m)^n = a^{mn}$; $\frac{a^{n+1}}{a^n} = a$; $\frac{a^n}{a^{n-1}} = a$; $a^2 - b^2 = (a+b)(a-b)$;

$(a \pm b)^2 = a^2 \pm 2ab + b^2$; $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$; $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$;

$(a + b + c)^2 = a^2 + 2ab + b^2 + 2ac + 2bc + c^2$.

Rod: $\sqrt[n]{a^n} = a$; $\sqrt[n]{a^{mn}} = a^m$; $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$; $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$; $\sqrt[n]{\frac{1}{a}} = \frac{1}{\sqrt[n]{a}} = a^{-\frac{1}{n}}$; $\sqrt[n]{a^m} = a^{\frac{m}{n}}$;

$\sqrt[n]{\sqrt[m]{a}} = \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$; $\sqrt{a^2} = \pm a$; $\sqrt{(a+b)^2} = \pm(a+b)$.

Ligning af 2. grad: $x^2 \pm px \pm q = 0$; $x = \mp \frac{p}{2} \pm \sqrt{\frac{p^2}{4} \mp q}$.

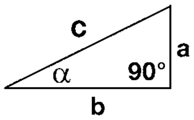
Aritmetisk række (differensrække):

$$a + (a + b) + (a + 2b) + (a + 3b) + \ldots.[a + (n - 1)b] = \left[a + \frac{n - 1b}{2}\right] n.$$

Geometrisk række (kvotientrække):

$$a + aq + aq^2 + aq^3 + \ldots.\, aq^{n-1} \frac{a(q^n - 1)}{q - 1}.$$

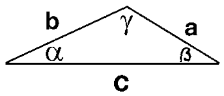
Trekantberegning



Retvinklet trekant:

$$\sin\alpha = \frac{a}{c};\quad \cos\alpha = \frac{b}{c};\quad \text{tg}\,\alpha = \frac{a}{b};\quad \text{ctg}\,\alpha = \frac{b}{a}$$

$$a^2 + b^2 = c^2.$$



Skævvinklet trekant:

$$a : b : c = \sin\alpha : \sin\beta : \sin\gamma \text{ eller } \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma};\quad \cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$a = c\cos\beta + b\cos\gamma = c\frac{\sin\alpha}{\sin\gamma}$$

$$c = a\cos\beta + b\cos\alpha = a\frac{\sin\gamma}{\sin\alpha}$$

$$b = a\cos\gamma + c\cos\alpha = a\frac{\sin\beta}{\sin\alpha}$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\sin\beta = \frac{b\sin\alpha}{a};\quad c = \frac{a\sin\gamma}{\sin\alpha};\quad c = b\cos\alpha \pm \sqrt{a^2 - b^2\sin^2\alpha}$$

$$\text{tg}\frac{a}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}},\quad \text{hvor } s = \frac{a+b+c}{2}.$$

Trigonometriske formler

$\sin^2\alpha + \cos^2\alpha = 1$

$\text{tg}\,\alpha = \frac{\sin\alpha}{\cos\alpha}$; $\text{ctg}\,\alpha = \frac{\cos\alpha}{\sin\alpha}$

$\text{tg}\,\alpha\,\text{ctg}\,\alpha = 1$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\text{tg}(\alpha \pm \beta) = \frac{\text{tg}\,\alpha \pm \text{tg}\,\beta}{1 \mp \text{tg}\,\alpha\,\text{tg}\,\beta}$

$\text{ctg}(\alpha \pm \beta) = \frac{\text{ctg}\,\alpha\,\text{ctg}\,\beta \mp 1}{\text{ctg}\,\beta \pm \text{ctg}\,\alpha}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta$

$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos\alpha\sin\beta$

$\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2\sin\alpha\sin\beta$

$\sin(45° \pm \alpha) = \cos(45° \mp \alpha)$

$\text{tg}(45° \pm \alpha) = \text{ctg}(45° \mp \alpha)$

$\text{tg}\,2\alpha = \frac{2\,\text{tg}\,\alpha}{1 - \text{tg}^2\alpha} = \frac{2}{\text{ctg}\,\alpha - \text{tg}\,\alpha}$

$\text{ctg}\,2\alpha = \frac{\text{ctg}^2\alpha - 1}{2\,\text{ctg}\,\alpha} = \frac{1}{2}(\text{ctg}\,\alpha - \text{tg}\,\alpha)$

$\sin\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}}$

$\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}}$

$\text{tg}\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$

$\text{ctg}\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}}$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$

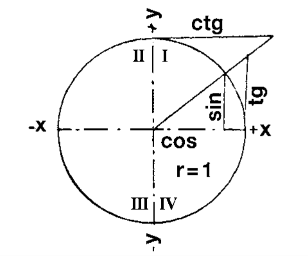
$\sin\alpha - \sin\beta = 2\cos\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$

Trigonometriske funktioner

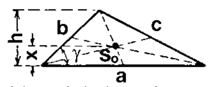
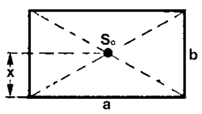
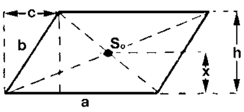
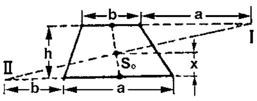
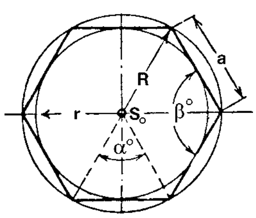
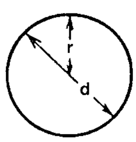
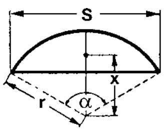
grafisk fremstilling og grænseværdier



Kvadrant	I	II	III	IV	Grad	0	30	45	60	90	180	270	360
sin	+	+	–	–	sin	0	$+\frac{1}{2}$	$+\frac{1}{2}\sqrt{2}$	$+\frac{1}{2}\sqrt{3}$	+1	0	–1	0
cos	+	–	–	+	cos	+1	$+\frac{1}{2}\sqrt{3}$	$+\frac{1}{2}\sqrt{2}$	$+\frac{1}{2}$	0	–1	0	+1
tg	+	–	+	–	tg	0	$+\frac{1}{3}\sqrt{3}$	+1	$+\sqrt{3}$	∞	0	∞	0
ctg	+	–	+	–	ctg	∞	$+\sqrt{3}$	+1	$+\frac{1}{3}\sqrt{3}$	0	∞	0	∞

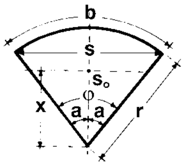
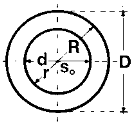
Flader

areal og tyngdepunkt

Betegnelse	Areal	Tyngdepunkt
Trekant  h = højden vinkelret på a $s = \frac{1}{2}(a + b + c)$	$F = \frac{1}{2}ah$ $= \frac{1}{2}ab\ \sin\gamma$ $= \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$	$x = \frac{h}{3}$ S_0 ligger i medianernes skæringspunkt
Rektangel 	$F = a \cdot b$	$x = \frac{b}{2}$ S_0 ligger i diagonalernes skæringspunkt
Parallelogram 	$F = ah = \cdot \sqrt{b^2 - c^2}$	$x = \frac{h}{2}$ S_0 ligger i diagonalernes skæringspunkt
Trapez 	$F = \frac{1}{2}\,h\,(a + b)$	$x = \frac{1}{3}h\,\frac{a + 2b}{a + b}$ Konstruktionen af S_0 fremgår af tegningen
Regelmæssig *n*-kant 	$F = \frac{na^2}{4}\ \text{ctg}\ \frac{\alpha}{2}$ $= \frac{nR^2}{2}\ \sin\alpha$ $= nr^2\ \text{tg}\ \frac{\alpha}{2}$	S_0 ligger i centrum
Cirkel  Periferien 0 = 2 π r	$F = \pi r^2$ $= \frac{\pi}{4}\,d^2$	S_0 ligger i centrum
Cirkelafsnit 	$F = \frac{r^2}{2}\left(\frac{\alpha \cdot \pi}{180} - \sin\alpha\right)$	$x = \frac{S^3}{12\,F}$

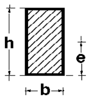
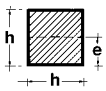
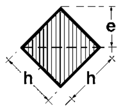
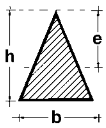
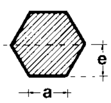
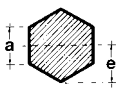
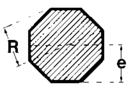
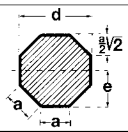
Flader, *fortsat*

areal og tyngdepunkt

Betegnelse	Areal	Tyngdepunkt
Cirkeludsnit 	$F = \frac{1}{2} br = \frac{\varphi°}{360} r^2\pi$ $\left(b = \pi r \frac{\varphi°}{180} = 0{,}01745 \; r\varphi°\right)$	$x = \frac{2}{3} r \frac{s}{b}$ for $\varphi = 60°$ $x = \frac{2r}{\pi} = 0{,}6366 \, r$ for $\varphi = 90°$ $x = \frac{4}{3} \sqrt{2} \frac{r}{\pi} = 0{,}6002 \, r$ for $\varphi = 180°$ $x = \frac{4}{3} \frac{r}{\pi} = 0{,}4244 \, r$
Cirkelring 	$F = \pi (R^2 - r^2)$ $= \frac{\pi}{4} (D^2 - d^2)$	S_0 ligger i centrum

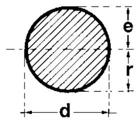
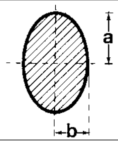
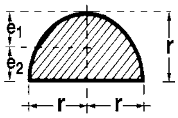
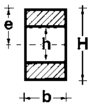
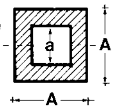
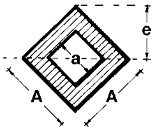
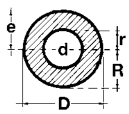
Tværsnit

areal, tyngdepunkt, inerti- og modstandmoment

Tværsnit	Areal F	Tyngdepunktets afstand e	Inertimoment J	Modstandsmoment $W = \frac{J}{e}$
	bh	$\frac{h}{2}$	$\frac{bh^3}{12}$	$\frac{bh^2}{6}$
	h^2	$\frac{h}{2}$	$\frac{h^4}{12}$	$\frac{h^3}{6}$
	h^2	$\frac{h}{2} \sqrt{2}$	$\frac{h^4}{12}$	$\frac{\sqrt{2}}{12} h^3 = 0{,}1179 \, h^3$
	$\frac{bh}{2}$	$\frac{2}{3} h$	$\frac{bh^3}{36}$	$\frac{bh^2}{24}$
	I en ligesidet trekant bliver $h = 0{,}8660 \, b$			
	$0{,}4333 \, b^2$	$0{,}5778 \, b$	$\sim 0{,}0181 \, b^4$	$\sim 0{,}0313 \, b^3$
	$\frac{3\sqrt{3}\, a^2}{2} = 2{,}598 \, a^2$	$a \sqrt{\frac{3}{4}} = 0{,}866 \, a$	$\frac{5\sqrt{3}}{16} a^4 = 0{,}5413 \, a^4$	$\frac{5}{8} a^3$
		a		$\frac{5\sqrt{3}}{16} a^3 = 0{,}5413 \, a^3$
	$2{,}828 \, R^2$	$0{,}924 \, R$	$\frac{1+2\sqrt{2}}{6} R^4$ $= 0{,}6381 \, R^4$	$0{,}6906 \, R^3$
	$0{,}8284 \, d^2$	Sidelængde $a = \frac{d}{1+\sqrt{2}} = 0{,}4142 \, d$	$0{,}0547 \, d^4$	$0{,}1095 \; d^3$

Tværsnit, *fortsat*

areal og tyngdepunkt, inerti- og modstandmoment

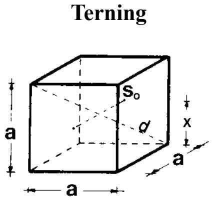
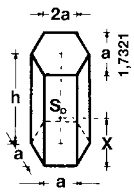
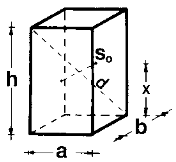
Tværsnit	Areal F	Tyngdepunktets afstand e	Inertimoment J	Modstandsmoment $W = \frac{J}{e}$
	$\pi r^2 = \frac{\pi d^2}{4}$	$\frac{d}{2}$	$\frac{\pi d^4}{64} = \frac{\pi r^4}{4}$ $= 0{,}0491\, d^4 \sim 0{,}05\, d^4$ $= 0{,}7854\, r^4$	$\frac{\pi d^3}{32} = \frac{\pi r^3}{4}$ $= 0{,}0982\, d^3 \sim 0{,}1\, d^3$ $= 0{,}7854\, r^3$
	πab	a	$\frac{\pi}{4}\, ba^3 = 0{,}7854\, ba^3$	$\frac{\pi}{4}\, ba^2 = 0{,}7854\, ba^2$
	$\frac{\pi}{2}\, r^2$	$e_1 = 0{,}4244\, r$ $e_2 = 0{,}5756\, r$	$= 0{,}1098\, r^4$	$W_1 = 0{,}2587\, r^3$ $W_2 = 0{,}1908\, r^3$
	$b\,(H - h)$	$\frac{H}{2}$	$\frac{b}{12}\,(H^3 - h^3)$	$\frac{b}{6H}\,(H^3 - h^3)$
	$A^2 - a^2$	$\frac{A}{2}$	$\frac{A^4 - a^4}{12}$	$\frac{1}{6}\,\frac{A^4 - a^4}{A}$
	$A^2 - a^2$	$\frac{A}{2}\sqrt{2}$	$\frac{A^4 - a^4}{12}$	$\frac{A^4 - a^4}{12A}\sqrt{2}$ $= 0{,}1179\, \frac{(A^4 - a^4)}{A}$
	$\frac{\pi}{4}\,(D^2 - d^2)$	$\frac{D}{2}$	$\frac{\pi}{64}\,(D^4 - d^4)$ $= \frac{\pi}{4}\,(R^4 - r^4)$	$\frac{\pi}{32}\,\frac{D^4 - d^4}{D}$ $= \frac{\pi}{4}\,\frac{(R^4 - r^4)}{R}$

Legemer

Volumen, overflade, tyngdepunkt m.m.

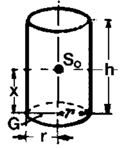
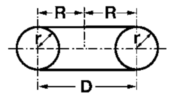
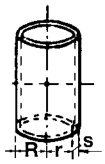
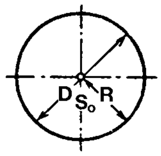
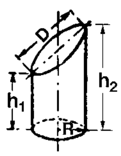
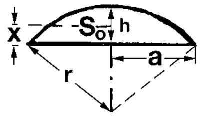
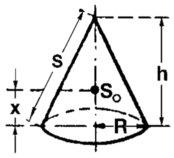
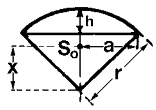
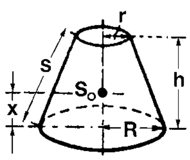
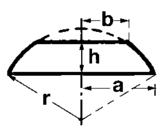
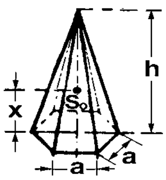
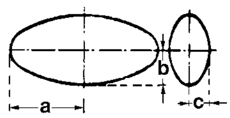
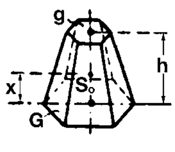
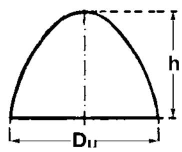
V = volumen; O = overflade; M = lodret eller skråtstillet overflade;

x = tyngdepunktets afstand fra grundfladen.

Terning 	$V = a^3$ $O = 6a^2$ $x = \frac{a}{2}$	$d = a\sqrt{3}$ $= 1{,}7321\, a$	**Sekssidet prisme** 	$V = 2{,}598\, a^2\, h$ $O = 5{,}1963\, a^2 + 6ah$ $x = \frac{h}{2}$ $d = \sqrt{h^2 + 4a^2}$ (jf. figuren for retv. prisme)
Retvinklet prisme 	$V = abh$ $O = 2\,(ab + ah + bh)$ $x = \frac{h}{2}$ $d = \sqrt{a^2 + b^2 + h^2}$		**Prisme** med regulær mangekant-grundflade G = grundflade a = sidelængde n = sidetal	$V = Gh$ $O = 2G + nha$ $M = nha$ $x = \frac{h}{2}$

Legemer, *fortsat*

volumen, overflade, tyngdepunkt m.m.

Cylinder 	$V = r^2\pi h = Gh$ $O = 2\pi r\,(r + h)$ $M = 2\pi rh$ $x = \frac{h}{2}$	**Ring med cirk. tv.** 	$V = 2\pi^2 Rr^2$ $= {}^1/_4\,\pi^2 Dd^2$ $O = 4\pi^2 Rr$ $= \pi^2 Dd$
Cylinderkappe (rør) 	$V = \pi h\,(R^2 - r^2)$ $= \pi s\,(2R - s)h$ $= \pi hs\,(2r + s)$ $x = \frac{h}{2}$	**Kugle** 	$V = \frac{4\pi r^3}{3} = \frac{\pi d^3}{6}$ $O = 4\pi r^2 = \pi d^2$ $r = \sqrt{\frac{3V}{4\pi}}$
Afskåret cylinder 	$V = R^2\pi\,\frac{h_1 + h_2}{2}$ $M = R\pi\,(h_1 + h_2)$ $D = \sqrt{4R^2 + (h_2 - h_1)^2}$ $x = \frac{h_1 + h_2}{2}\cdot\left(1 + \frac{(h_2 - h_1)^2}{4(h_2 + h_1)^2}\right)$	**Kuglekalot** 	$V = \frac{\pi h}{6}(3a^2 + h^2)$ $= \frac{\pi h^2}{3}\,(3h - h)$ $M = 2\pi rh = \pi\,(a^2 + h^2)$ $a^2 = h\,(2r - h)$ $x = \frac{3}{4}\cdot\frac{(2r - h)^2}{3r - h}$
Kegle 	$V = \frac{\pi R^2 h}{3};\ x = \frac{h}{4}$ $M = \pi Rs$ $s = \sqrt{R^2 + h^2}$	**Kugleudsnit** 	$V = \frac{2\pi r^2 h}{3}$ $O = \pi r\,(2h + a)$ $x = \frac{3}{8}(2r - h)$
Keglestub 	$V = \frac{\pi h}{3}\,(R^2 + Rr + r^2)$ $= \frac{h}{4}\left[\pi\sigma^2 + \frac{1}{3}\pi\delta^2\right]$ $M = \pi s\sigma$ $\sigma = R + r \quad \sqrt{\delta^2 + h^2}$ $\delta = R - r$ $x = \frac{h}{4}\cdot\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}$	**Kuglebælte** 	$V = \frac{\pi h}{6}(3a^2 + 3b^2 + h^2)$ $M = 2\pi rh$ $r^2 = a^2 + \left(\frac{a^2 - b^2 - h}{2h}\right)^2$
Pyramide 	$V = \frac{Gh}{3}$ $x = \frac{h}{4}$	**Ellipsoide** 	$V = \frac{4}{3}\pi\,abc$
Pyramidestub 	$V = \frac{h}{3}(G + g + \sqrt{Gg})$ $x = \frac{h}{4}\cdot\frac{G + 2\sqrt{Gg} + 3g}{G + \sqrt{Gg} + g}$	**Omdrejningsparaboloide** 	D_u = underfladens diameter $V = \frac{d_u^2\cdot\pi}{4}\cdot\frac{h}{2}$